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ULTIMATE KINEMATIC CHARACTERISTICS OF COMPOSITE SOLIDS ACCELERATED BY A MAGNETIC FIELD

S. V. Stankevich and G. A. Shvetsov

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The present paper considers the ultimate (under heating conditions) kinematic characteristics of composite solid bodies accelerated by unsteady magnetic-field pressure. The accelerated sheet comprises two layers: a layer of a composite material consisting of a mixture of two materials with different electrothermal properties and a homogeneous material layer. The electrical properties of the composite layer with the coordinate are varied along the coordinate by changing the volume concentration of its constituent materials. For an exponential magnetic field rise, an analytical solution is obtained for the problem of determining the optimum distribution of the volume concentration of the composite constituents for which there is a maximum increase in the ultimate velocity of the sheet. Numerical simulation showed that the distribution of the volume concentration obtained from analytical relations is also nearly optimal for pulse shapes of the accelerating magnetic field different from exponential ones. The possibility of considerably increasing the ultimate velocity through the use of composite layers compared to the ultimate velocities for the homogeneous materials constituting the composite is shown analytically and numerically.

Introduction. One factor limiting maximum velocities during electromagnetic acceleration of solids is the heating of conductors by the currents flowing in them to temperatures above the melting point of the material. This can lead to loss of the mechanical strength of the conductors, change of their shape, and ultimately failure. The requirement of no melting of conductors during acceleration imposes restrictions on the maximum permissible amplitudes of the accelerating magnetic fields, thus limiting the maximum velocity to which a conductor of given mass can be accelerated [1].

In some papers (see, e.g., [1–4]), it was noted that the use of heterogeneous conductors with electrical conductivity increasing discretely or continuously with distance from the surface can decrease their local heating considerably.

In this connection, it is of interest to determine the ultimate kinematic characteristics of sheets containing a layer with electrical conductivity increasing continuously in the direction of magnetic-field diffusion. The optimum law of variation of electrical conductivity in this layer can be found from the natural condition that at the end of acceleration, the temperature at each point of this layer reaches a certain critical value. A similar formulation of the problem of increasing critical magnetic field by using composite materials was considered in [2], where an optimum profile of electrical conductivity variation was obtained for a layer in contact with a conducting homogeneous half-space. In this paper, however, the heat capacity (in contrast to electrical conductivity) was considered constant at each point of the layer. Generally, variation of electrical conductivity, for example, by changing the volume concentration of well- and poorly conducting particles results in variation of the averaged thermal properties of the material from point to point. In addition, unlike in problems related to magnetic-field generation, in problems of electromagnetic acceleration of conductors, it is necessary to take into account the finite dimensions of the conductors and the variation of the average density of the material and to specify the amplitude and duration of the accelerating magnetic field so that the maximum velocity is reached at a specified acceleration distance.

Lavrent'ev Institute of Hydrodynamics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090.
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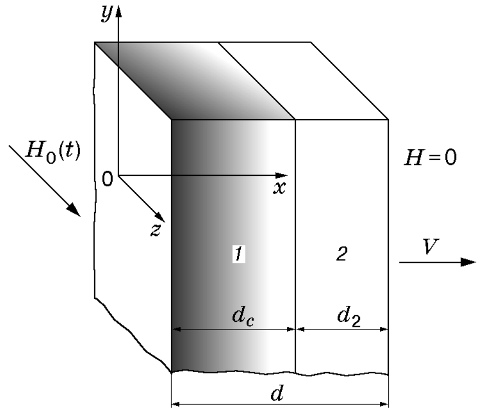

Fig. 1

Formulation of the Problem. We consider the acceleration of a conducting flat sheet by unsteady magnetic field pressure. On the sheet surface, the time dependence of the magnetic field $H_0(t)$ is given by the relation

$$H_0(t) = H_s h_0(\tau) \qquad (\tau = t/t_0),$$

where H_s and t_0 are arbitrary parameters that specify the amplitude and characteristic duration of the accelerating magnetic field pulse and $h_0(\tau)$ is an arbitrary function. Considering that at the initial time, the velocity of the sheet $V = 0$, the velocity and the distance L at an arbitrary time can be obtained from the relations

$$V = \frac{\mu H_s^2 t_0}{2\bar{\rho} d}\, v(\tau), \qquad L = \frac{\mu H_s^2 t_0^2}{2\bar{\rho} d}\, l(\tau), \tag{1}$$

where $v(\tau) = \int\limits_0^\tau h_0^2(\tau)\, d\tau$, $l(\tau) = \int\limits_0^\tau v(\tau)\, d\tau$, and $\bar{\rho} = \frac{1}{d}\int\limits_0^d \rho(x)\, dx$ are the dimensionless velocity, acceleration distance, and the average density of the sheet, respectively.

Generally, we assume that the accelerated sheet of thickness d comprises two layers in contact: a composite layer of thickness d_c consisting of a mixture of two materials (first and second) with different electrothermal properties and a homogeneous layer of thickness d_2 made of the second material (Fig. 1). Below, the subscript and superscript 1 and 2 denote parameters of the first and second materials, respectively. Let the electrical conductivity of the second material be higher than the electrical conductivity of the first material ($\sigma_2 > \sigma_1$) and let the electrical conductivity at points of the composite layer be changed as a result of change in the volume concentration $\varepsilon(x)$ of the second material (the x coordinate is reckoned from the sheet surface in contact with the field). Furthermore, the characteristic sizes of the composite particles are small so that it is possible to ignore the variations of the magnetic and thermal fields due to the discrete dependence of the electrothermal properties of the composite material on the coordinates. We note that the jumps of current density and Joule heat release at the interfaces between particles with different electrical conductivity can remain finite for any particle sizes, but a decrease in particle size due to heat conduction ensures local temperature equalizing. Thus, the averaged properties of the composite material are assumed to depend continuously on the x coordinate according to the volume concentration distribution $\varepsilon(x)$. In this case, at the interface between the composite and homogeneous layers, we set $\varepsilon = 1$.

For an arbitrary composite material, the density ρ and heat capacity C per unit volume can be obtained from the relations

$$\rho(x) = \rho_1(1 - \varepsilon(x)) + \rho_2\varepsilon(x), \qquad C(x) = \rho_1 c_1(1 - \varepsilon(x)) + \rho_2 c_2 \varepsilon(x). \tag{2}$$

At the same time, the dependence of the averaged electrical conductivity σ on the volume concentration ε can be determined only for a composite material of known structure or experimentally. Below, we assume that the dependence $\sigma(\varepsilon)$ is known.

We ignore the temperature dependence of electrothermal properties, the compressibility of the materials, and heat transfer between the sheet and the ambient medium. Then, the magnetic-field and temperature distributions in the sheet can be obtained by solving the following magnetic-field diffusion and heat-conduction equations:

$$\lambda^2 \frac{\partial h}{\partial \tau} = \frac{\partial}{\partial \xi} \frac{1}{\tilde{\sigma}} \frac{\partial h}{\partial \xi}, \qquad \lambda^2 \tilde{C} \frac{\partial \theta}{\partial \tau} = \gamma \frac{\partial}{\partial \xi} \tilde{k} \frac{\partial \theta}{\partial \xi} + \frac{2}{\tilde{\sigma}} \Big(\frac{\partial h}{\partial \xi}\Big)^2 \tag{3}$$

subject to the initial and boundary conditions

$$\theta(\xi, 0) = 0, \quad h(\xi, 0) = 0, \quad h(0, t) = h_0(\tau), \quad h(\xi_d, t) = 0, \quad \frac{\partial \theta}{\partial \xi}\Big|_{\xi=0} = \frac{\partial \theta}{\partial \xi}\Big|_{\xi=\xi_d} = 0. \tag{4}$$

In Eqs. (3) and (4), $\tau = t/t_0$, $\xi = x/x_s$, $h = H/H_s$, $\theta = 2\rho_2 c_2 \Delta T/(\mu H_s^2)$, $\tilde{\rho} = \rho/\rho_2$, $\tilde{C} = C/(\rho_2 c_2)$, $\tilde{\sigma} = \sigma/\sigma_2$ are dimensionless variables, $\xi_d = d/x_s$, $\gamma = \mu\sigma_2 k_2/(\rho_2 c_2)$, $\lambda^2 = \mu\sigma_2 x_s^2/t_0$, and $\Delta T = T - T_0$ is the temperature variation in the sheet (T_0 is the initial temperature and k_2 is the thermal conductivity; the length scale x_s can be chosen arbitrarily). Because for most conductors, $\gamma < 10^{-2}$, it follows that in a macrodescription of the Joule heating process, we can ignore heat conduction, setting $\gamma = 0$ in the second equation of system (3).

Let T_* be the lowest among the melting points of the materials constituting the composite layer. It is required to find the dependence $\varepsilon(\xi)$, the maximum possible field amplitude H_s, the duration of the action t_0, and the thickness of the composite layer d_c for which a sheet of thickness d has maximum velocity at specified acceleration distance L provided that the temperature at each point of the sheet does not exceed T_* during the entire acceleration time. In the case considered, using the distribution $\theta(\xi, \tau, \lambda)$ obtained from the solution of system (3) and the definition of the dimensionless temperature θ (5), we can obtain the maximum possible value of H_s for which the maximum temperature at a certain point of the sheet reaches T_* at an arbitrary time τ:

$$H_s = \sqrt{2\Delta Q_*/(\mu\theta_{\max}(\lambda, \tau))}. \tag{5}$$

Here $\Delta Q_* = \rho_2 c_2 (T_* - T_0)$ and the maximum dimensionless temperature $\theta_{\max}(\lambda, \tau)$ is defined by relation

$$\theta_{\max}(\lambda, \tau) = \max_{\substack{0 \leqslant \xi \leqslant \xi_d \\ 0 \leqslant \tau' \leqslant \tau}} \theta(\lambda, \xi, \tau'). \tag{6}$$

Substitution of the magnetic-field amplitude (5) into the kinematic relations (1) yields

$$\frac{V}{d} = \frac{\mu\sigma_2 \Delta Q_*}{\bar{\rho}} \frac{v(\tau)}{(\lambda\xi_d)^2 \theta_{\max}(\lambda, \tau)}, \qquad \frac{L}{d^3} = \frac{(\mu\sigma_2)^2 \Delta Q_*}{\bar{\rho}} \frac{l(\tau)}{(\lambda\xi_d)^4 \theta_{\max}(\lambda, \tau)}. \tag{7}$$

The dependence of the ultimate sheet velocity on sheet thickness in parametric form for any acceleration distance can be found from relations (7) using solutions of Eqs. (3) for several values of the parameter λ.

Because the parameters on the left and right sides of Eqs. (7) are independent of each other, the dependence $V(d)$ obtained for an acceleration distance L can be converted to the dependence $V'(d')$ for an acceleration distance $L' = aL$ using the transformations $V' = a^{1/3} V$ and $d' = a^{1/3} d$.

Analytical Solution. The optimum distribution of the volume concentration $\varepsilon(\xi)$ that ensures uniform heating can be obtained in analytical form using the steady-state solutions of system (3) admissible for $h_0(\tau) = e^{\tau}$. The steady-state solutions are close to the true solutions (obtained subject to the initial conditions) if the acceleration time far exceeds the characteristic times of the transient processes involved in the establishment of the magnetic field, i.e., for $\tau \gg 1$. If in (3) we set $h(\xi, \tau) = e^{\tau} h(\xi)$, $\theta(\xi, \tau) = e^{2\tau}\theta(\xi)$, and $\gamma = 0$, we obtain equations for steady-state distributions of temperature and magnetic field in the sheet. The magnetic-field distribution in the sheet can be obtained by solving the boundary-value problem

$$(h'(\xi)/\tilde{\sigma})' = \lambda^2 h(\xi), \qquad h(0) = 1, \quad h(\xi_d) = 0, \tag{8}$$

and the temperature distribution is given by the expression

$$\theta = (h'(\xi)/\lambda)^2/(\tilde{\sigma}\tilde{C}). \tag{9}$$

Here and below, the prime denotes differentiation with respect to the variable ξ. If the temperature in the composite layer has the same value θ_c for any $\xi \leqslant \xi_c$, then, according to (9),

$$h'(\xi) = \lambda\sqrt{\theta_c}\,\sqrt{\tilde{\sigma}\tilde{C}}. \tag{10}$$

Substitution of this expression into (8) yields

$$h(\xi) = -\sqrt{\theta}_c y'/\lambda, \tag{11}$$

where $y = \sqrt{\tilde{C}/\tilde{\sigma}}$ is an unknown function. Differentiation of (11) using (10) gives

$$y'' = \lambda^2 y\tilde{\sigma}. \tag{12}$$

Equation (12) can be used to find $y(\xi)$ if the dependence $\tilde{\sigma}(y)$ is known. Using the dimensionless expression for the volumetric heat capacity (2), we obtain

$$[\varepsilon + \tilde{C}_1(1-\varepsilon)]/\tilde{\sigma}(\varepsilon) = y^2. \tag{13}$$

Solving this equation with respect to ε (it is assumed that this can be done for a certain range of ε), we obtain the dependence $\varepsilon(y^2)$, and, hence, the dependences $\varepsilon(y^2)$, $\tilde{\sigma}(y^2)$, $\tilde{C}(y^2)$, and $\tilde{\rho}(y^2)$. Thus, for example, if the composite layer has a layered or fibrous structure (the direction of the fibers coincides with the direction of the current), then, $\tilde{\sigma} = \varepsilon + (1-\varepsilon)\tilde{\sigma}_1$. In this case, we obtain

$$\varepsilon(y^2) = \frac{\tilde{C}_1 - y^2\tilde{\sigma}_1}{y^2(1-\tilde{\sigma}_1) + \tilde{C}_1 - 1}, \qquad \tilde{\sigma} = \frac{\tilde{C}_1 - \tilde{\sigma}_1}{y^2(1-\tilde{\sigma}_1) + \tilde{C}_1 - 1}. \tag{14}$$

Using the solution of Eq. (8) for the magnetic-field distribution in the homogeneous layer and taking into account the continuity condition for the electric and magnetic fields at the interface between the composite and homogeneous layers, we obtain the first integral of Eq. (12):

$$y'^2 = \lambda^2\left(\int_1^{y^2} \tilde{\sigma}(y^2)\,dy^2 + \frac{1}{\theta^0_{\max}(\lambda\xi_2)}\right). \tag{15}$$

Here $\theta^0_{\max}(\lambda\xi_2) = (\cosh\lambda\xi_2/\sinh\lambda\xi_2)^2$ is the maximum temperature of the homogeneous layer.

Substituting the expression for the derivative of the function y (15) into (11) and using the boundary condition $h(0) = 1$, we obtain the dimensionless temperature in the composite layer:

$$\theta_c(y_0^2, \lambda\,\xi_2) = \left(\frac{\lambda}{y'}\right)^2 = \left(\int_1^{y_0^2} \tilde{\sigma}(y^2)\,dy^2 + \frac{1}{\theta^0_{\max}(\lambda\xi_2)}\right)^{-1}, \tag{16}$$

where $y_0 = y|_{\xi=0}$.

Substituting the expression of electrical conductivity for the composite layer (14) into the integral entering into (16), we obtain

$$\theta_c(y_0^2, \lambda\xi_2) = \left(\frac{\tilde{C}_1 - \tilde{\sigma}_1}{1-\tilde{\sigma}_1} \ln\left(\frac{y^2(1-\tilde{\sigma}_1) + \tilde{C}_1 - 1}{\tilde{C}_1 - \tilde{\sigma}_1}\right) + \frac{1}{\theta^0_{\max}(\lambda\,\xi_2)}\right)^{-1}. \tag{17}$$

We note that formulas (16) and (17) are also valid for sheets consisting only of a composite layer ($\xi_2 = 0$ and $1/\theta^0_{\max}(\lambda\xi_2) = 0$) and in the case of homogeneous layers of infinite dimensions ($\xi_2 \to \infty$ and $1/\theta^0_{\max}(\lambda\xi_2) = 1$). For a homogeneous layer of finite dimensions, we can set $x_s = d_2$. Then, $\xi_2 = 1$ and $\xi_d = \xi_c + 1$.

Separation of the variables in (15) and integration yield the dependence $y(\xi)$ in the form of the inverse function

$$\xi(y, y_0, \lambda\xi_2) = \int_y^{y_0} \frac{dy}{y'(y, \lambda\xi_2)}. \tag{18}$$

The thickness of the composite layer can be determined by setting in (18) $y = 1$ [$\xi_c(y_0, \lambda\xi_2) = \xi(1, y_0, \lambda\xi_2)$]. The volume-concentration distribution for the first material in the composite layer $\varepsilon(\xi,\, y_0)$ is given in parametric form by relations (13) and (18) with y varying from 1 to y_0.

Using (18), we obtain the average density of the sheet:

$$\bar{\rho}(y_0, \lambda\xi_2) = \frac{1}{\xi_d}\int_0^{\xi_d} \rho\,d\xi = \frac{\rho_2}{\xi_c + \xi_2}\left(\int_1^{y_0} \frac{\rho(y)\,dy}{y'(y, \lambda\xi_2)} + \xi_2\right). \tag{19}$$

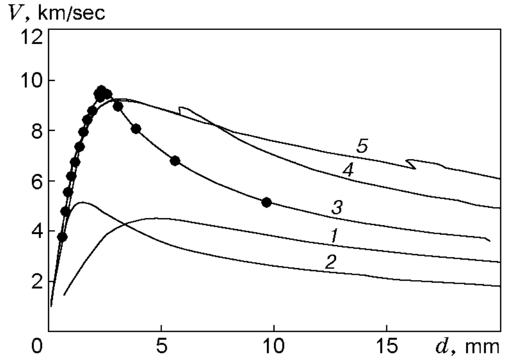

Fig. 2.

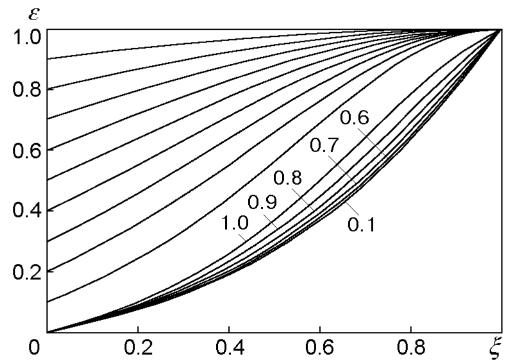

Fig. 3.

In the homogeneous layer, the maximum temperature is reached at the interface with the composite layer. Since the function $\tilde{\sigma}$ is continuous at the interface between the layers (at the interface, $\varepsilon = 1$), the temperature θ also remains a continuous function at this interface. Consequently, θ_c is the maximum temperature in the sheet and, according to the definition (6), $\theta_{\max} = \theta_c e^{2\tau}$.

Substituting the expressions for the maximum temperature $\theta_{\max}$ into the kinematic relations (7) taking into account (16), the average density of the sheet (19) and the dimensions of the sheet (18), and $v(\tau) = e^{2\tau}/2$ and $l(\tau) = e^{2\tau}/4$, we obtain the dependence of the ultimate velocity of the sheet $V(y_0, \lambda\xi_2)$ on its thickness $d(y_0, \lambda\xi_2)$ in parametric form

$$V(y_0, \lambda\xi_2) = \Big(\frac{\mu\sigma_2 L}{2}\Big(\frac{\Delta Q_*}{\bar{\rho}(y_0, \lambda\xi_2)\lambda\xi_d(y_0, \lambda\xi_2)\theta_c(y_0, \lambda\xi_2)}\Big)^2\Big)^{1/3},$$
$$d(y_0, \lambda\xi_2) = \Big(\frac{4L\bar{\rho}(y_0, \lambda\xi_2)\lambda^4\xi_d^4(y_0, \lambda\xi_2)\theta_c(y_0, \lambda\xi_2)}{(\mu\sigma_2)^2\Delta Q_*}\Big)^{1/3}. \tag{20}$$

An analysis of relations (16)–(20) shows that for a sheet consisting of a composite and homogeneous layers ($\xi_2 > 0$), the dependence $V(d)$ ($L = \text{const}$) can be obtained by varying the parameter λ from 0 to ∞ for any value of y_0 ($1 \leqslant y_0 \leqslant \sqrt{\tilde{C}_1/\tilde{\sigma}_1}$) admissible for a given pair of materials but the maximum velocity for a sheet with fixed d is attained for the maximum value $y_{\max} = \max\limits_{0\leqslant\varepsilon\leqslant 1} y_0 = y_0\Big|_{\varepsilon=0} = \sqrt{\tilde{C}_1/\tilde{\sigma}_1}$, i.e., for zero concentration of the second material on the surface of the composite layer.

The curve of $V(d)$ for $\lambda \to 0$ ($\xi_2 > 0$ and $y_0 = y_{\max}$) begins at a certain point $V_0 = V(y_{\max}, 0)$, $d_0 = d(y_{\max}, 0) > 0$, determined from relations (20). At this point, the thickness of the composite layer is much greater than the thickness of the homogenous layer ($\xi_c/\xi_d \to 1$). For sheet dimensions $d \leqslant d_0$, the sheet consisting only of a composite layer ($\xi_2 = 0$) has maximum velocity. In this case, relations (20) do not depend on the parameter λ and the dependence $V(d)$ can be obtained for $1 \leqslant y_0 \leqslant y_{\max}$.

Figure 2 shows curves of ultimate velocity versus sheet thickness calculated using the above relations. Curves 3–5 refer to a sheet consisting of a composite layer (Fe + Cu) and a homogeneous layer (Cu). Curves 1 and 2 refer to homogeneous sheets (Cu and Fe, respectively). For curves 4 and 5, the electrical conductivity of iron is decreased by a factor of 10 and 100, respectively. The calculations were performed for the electrothermal parameters of the materials averaged over the temperature range from room temperature to the melting point of copper. Curves 3–5 have inflections at the points $d_0(y_{\max})$. The segments of the curves before the points of inflection correspond to the sheet consisting only of a composite layer. The segments of the curves behind the points of inflection correspond to the sheet consisting of a composite–homogeneous layer. On curve 3, the points correspond to a change in the volume concentration of copper by 0.1 for $d \leqslant d_0$ or to a change of the relative thickness of the composite layer $\nu = \xi_c/\xi_d$ by 0.1 for $d > d_0$. For a certain thickness of the sheet in the neighborhood of the point of inflection, up to three different distributions of $\varepsilon(x)$ can exist that ensure uniform heating. The maximum ultimate velocity of the sheet containing an optimized composite layer is about a factor of 2.1 and 2.3 higher than the maximum ultimate velocity for homogeneous sheets of copper and iron, respectively.

Figure 3 shows the optimum distributions of copper concentration in a composite layer consisting of copper and iron. These distributions correspond to the sheet thicknesses marked by points in Fig. 2. The curves with zero surface concentration correspond to the case of acceleration of sheets consisting of a composite layer and a homogeneous copper layer. The numbers in Fig. 3 correspond to the relative thickness of the composite layer. The curves with nonzero surface concentration of copper were obtained for a sheet consisting only of a composite layer. It should be noted that for composite layers of relative thickness $\nu < 0.7$, the optimum profiles of copper distribution in the composite layer practically do not differ from each other.

It follows from Fig. 3 that as the thickness of the sheet decreases, the composite layer becomes similar in properties to a homogeneous sheet of the material with higher electrical conductivity (second material). In this case, the curve of $V(d)$ (Fig. 2) approaches the asymptote to a "thin sheet" $V/d = \text{const}$ [5] for the second material. But the ultimate velocity of the composite sheet enters this asymptote for larger values of d than those for the homogeneous sheet. Accordingly, its maximum ultimate velocity can be much higher than the maximum ultimate velocity of the homogeneous sheet. With increase in the thickness of sheets containing a composite layer of any pair of materials, the ultimate velocity always decreases, even in the case of artificial decrease in the electrical conductivity of the first material (curves 4 and 5) in Fig. 2. However, for great d, the ultimate velocity of sheets containing a composite layer is higher than the ultimate velocity of homogeneous sheets made of the materials constituting the compact. With decrease in $\tilde{\sigma}_1$, the increase in ultimate velocity becomes more considerable. The ratio v_r of the ultimate velocity of a sheet with a composite layer to the ultimate velocity of a homogeneous sheet can be calculated for sheets having identical masses per unit areas and for identical acceleration distances.

For rather thick homogeneous sheets, the following asymptotic expression for the ultimate velocity is valid [1]:

$$V = \psi\sqrt{\Delta Q_p L/(\rho_p d_p)}. \tag{21}$$

Here ψ is a dimensionless parameter that depends only on the shape of the accelerating field pulse [for $h_0(\tau) = \mathrm{e}^\tau$, $\psi = 1$]; the subscript p refers to the parameters of the homogeneous sheet. Using relations (20) and setting in (21) $\rho_p d_p = \bar{\rho} d$, we have

$$v_r = \sqrt{\Delta Q_*/\Delta Q_p}\,\sqrt{1/\theta_c}.$$

We note that v_r does not depend on the mass of the sheet and acceleration distance. In a layered composite material, the quantity θ_c is given by expression (17). In this case, assuming that $\sigma_2 \gg \sigma_1$ and that for rather thick sheets, $1/\theta^0_{\max}(\lambda\xi_2) \approx 1$, we obtain

$$v_r = \sqrt{\Delta Q_*/\Delta Q_p}\,\sqrt{(\rho_1 c_1/(\rho_2 c_2))\ln(\sigma_2/\sigma_1) + 1}. \tag{22}$$

From this relation, we can estimate the increase in the ultimate velocity for a sheet containing an optimized composite layer of any two materials compared to the ultimate velocity for a homogeneous sheet of any of these materials. For a sheet containing a composite layer of iron and copper, we obtain the following ratios: $V_{(\mathrm{Fe+Cu})-\mathrm{Cu}}/V_{\mathrm{Cu}} = 1.95$ and $V_{(\mathrm{Fe+Cu})-\mathrm{Cu}}/V_{\mathrm{Fe}} = 1.38$. From (22) it follows that the velocity ratio v_r increases slightly with decrease in the electrical conductivity of the first material constituting the composite layer. The electrical conductivity of the materials can be decreased by the addition of small amounts of various dopes without significant changes in the density and heat capacity of the composite material. Thus, if the electrical conductivity of iron is decreased by a factor of 100, the ratios given above are equal to 3.23 and 2.28, respectively. From formula (22) it follows that $v_r \to \infty$ as $\sigma_1 \to 0$ [formula (26) is obtained for $d \to \infty$]. However, as follows from Fig. 2, for a sheet of fixed thickness, the relative increase in velocity is always finite.

Numerical Solution. The ultimate (for heating conditions) kinematic characteristics of the sheets were calculated by numerical solution of Eqs. (3) using the model of a layered composite layer in which the average electrical conductivity and the volume concentration of the high-conducting material are linked by relation (14). For a given pair of materials and a fixed relative thickness of the composite layer ν, we obtained the optimum distribution of the relative volume concentration of the well-conducting material in the composite layer $\varepsilon(\xi)$ and the corresponding dependences $\tilde{\rho}(\xi)$, $\tilde{c}(\xi)$, and $\tilde{\sigma}(\xi)$. By numerical calculations using the above dependences and a specified function $h_0(\tau)$ for several values of the parameter λ, we determined the maximum temperature in the sheet at the time of cessation of acceleration τ. Next, the ultimate velocity and sheet thickness were determined for a specified acceleration distance using the kinematic relations (7).

Calculations for an exponentially increasing accelerating magnetic field showed that the effect of the transient processes involved in the establishment of a steady-state field distribution in the sheet could be ignored for $\tau \geqslant 3$. In this case, the error of velocity calculations using the approximate analytical approach does not exceed 2%.

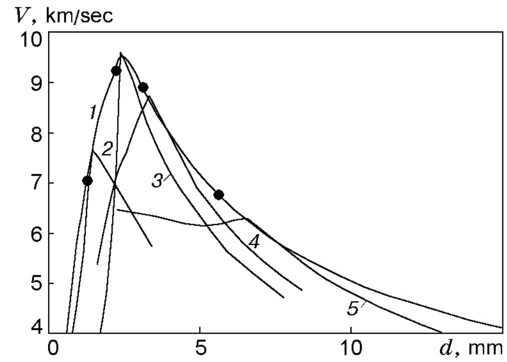

Fig. 4

The ultimate velocity was also calculated for the cases where the time variation of the accelerating field $h_0(\tau)$ was specified by the functions τ, $\sin(\pi\tau/2)$, and $\tau e^{1-\tau}$.

Figure 4 shows calculated ultimate velocity versus sheet thickness for a sheet containing a composite layer of copper and iron for an acceleration length of 0.1 m and $h_0(\tau) = \tau$. Curve 1 is obtained by analytical solution, and curves 2–5 are the results of numerical calculations for the different relative thicknesses of the optimized composite layer ν. Curve 2 refers to $\nu = 1$ for $\varepsilon(0) = 0.45$ and curves 3–5 refer 1, 0.5, and 0.3, respectively, for $\varepsilon(0) = 0$. The points on curve 1 correspond to sheets with the same values of ν and $\varepsilon(x/d)$ that were used in numerical calculations. The velocity corresponding to the maximum points on curves 2–5 is close the analytical velocity of the sheet (curve 1) but the maximum velocity is attained for sheet dimensions 5–15% larger than the dimensions obtained in analytical calculations. Calculations for $h_0(\tau) = \sin(\pi\tau/2)$ and $h_0(\tau) = \tau e^{1-\tau}$, and relative acceleration time $\tau = 1$, showed that the curves of ultimate velocity versus sheet thickness coincide with an accuracy of a few percent with curves 2–5 shown in Fig. 4, which were obtained for $h_0(\tau) = \tau$.

Conclusions. From the analysis performed it follows that the use of a composite material with electrical conductivity increasing in the direction of magnetic-field diffusion can increase the ultimate (for heating conditions) kinematic characteristics of accelerators. Thus, the ultimate velocity of a sheet containing a composite layer of iron and copper is about twice that of homogeneous sheets of iron and copper. When the electrical conductivity of iron decreases by a factor of 100, the ultimate velocity increases by a factor 3.

For a given pair of materials constituting a composite sheet, the optimum profile of distribution of the high-conducting material in the composite layer depends on the required sheet thickness and specified acceleration distance. A procedure for obtaining the optimum profile and structure of the sheet is described for the case of an exponentially increasing accelerating magnetic field.

Numerical simulation showed that the analytical optimum structure for a sheet of given thickness is also nearly optimal in the case where the time dependence of the magnetic field is specified by different functions [in particular, $h_0 = \tau$, $h_0 = \sin(\pi\tau/2)$, and $h_0(\tau) = \tau e^{1-\tau}$ for $\tau = 1$] but the sheet thickness is 5–15% greater.

The analysis showed that increasing the ratio of electrical conductivities of the materials constituting a composite sheet, one can achieve a considerable increase in ultimate velocity compared to homogeneous sheets. Interesting results can be obtained through the use of combinations of conducting and nonconducting materials. However, in this case, to ensure microuniform heating of the composite material, it is necessary to decrease the characteristic particle size in the composite material and(or) to use a material with high thermal conductivity as an insulator.

The analysis performed does not cover all aspects of the use of composite materials as current-carrying elements of magnetically driven projectiles. In particular, the thermomechanical and strength properties of the composite constituent materials should be chosen in a special manner to ensure the integrity of the projectile during acceleration.

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